The Limit Assumption in Deontic (and Prohaiaretic) Logic

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Preface

ANALYOMEN — Perspectives in Analytical Philosophy was GAP's first congress. GAP, in turn, is the *Gesellschaft für Analytische Philosophie*, founded in Berlin in 1990. *ANALYOMEN* took place at the *Universität des Saarlandes* in Saarbrücken, October 9—12, 1991. The congress and its selected proceedings are intended to exemplify, and to reflect upon, the present and future role and nature of analytic philosophy. Georg Henrik von Wright's opening lecture addressed these questions explicitly, and provided, at the same time, a survey of their historical background. In an open section, and [specialist] sections on logic, epistemology, philosophy of science, philosophy of language, philosophy of mind, and practical philosophy, philosophers from all over the world reported on what it is that they are doing, and how they do it.

ANALYOMEN's selected proceedings are "selected" in two completely different respects. First, some speakers themselves selected by deciding not to submit the papers they read at the conference. There are various reasons for this, and most of them have nothing to do with the quality of the material; one frequent reason, and a perfectly legitimate one, is that the material presented at *ANALYOMEN* had already been published, or submitted, elsewhere. Second, a jury, elected by GAP's managing committee, selected among the papers that were submitted.

ANALYOMEN's selected proceedings open de Gruyter's series *Perspectives in Analytic Philosophy/Perspektiven der Analytischen Philosphie*. A workshop on rational choice was part of *ANALYOMEN*, and its proceedings, *Praktische Rationalität*, edited by Julian Nida-Rümelin und Ulla Wessels, will also be published in this series.

ANALYOMEN also provided the context for GAP's first regular general meeting. An appendix to the present volume provides information on GAP's genesis, aims, and perspectives.

ANALYOMEN was subsidized by: *Deutsche Forschungsgemeinschaft*, *Wissenschaftsministerium des Saarlandes*, *Universität des Saarlandes*, *Freunde der Universität des Saarlandes*, and *Rechtsanwalt* Wilhelm Bick. The congress could not have taken place without these subsidies, and GAP is grateful to all these institutions and to Mr Bick.

ANALYOMEN benefited from many people's support: Christoph Fehige invested a considerable amount of good thinking, and many of his ideas have shaped the project; preparing the congress and running it smoothly would not have been possible without Barbara Kohl's energy and initiative; Klaus Peter Rippe contributed in numerous respects; Karina Bauer, Christoph Ernestus, Bernd Hoffmann, Frank Kiefer, Volker Schöpfer, Sebastian Varges und Claudia Villiger helped with the organization, especially during the days of the conference; in Saarbrücken, the programme could not have been run, and the schedule not obeyed, without the help of the chairs of the sections; various members of GAP as well as other participants came forward with helpful suggestions and initiatives; after the conference, the jury that had to select among the submitted papers faced a difficult task; many of the authors who are not English native speakers followed the editors' call for contributions written in English, and thus helped making this volume accessible to the international academic community. We are grateful to everybody for their support.

Georg Meggle & Ulla Wessels
Saarbrücken, July 1992

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The Limit Assumption in Deontic (and Prohairetic) Logic*

CHRISTOPH FEHIGE

What should the logic of "ought" say if confronted with feasible sets that don't have optimal elements? The problem has been known ever since the semantics of deontic logic was tied to normative preference relations, i.e. ever since Danielsson (1968). Writers who employ a semantics of that type usually mention the question and react to it one way or the other, cf. Åqvist (1987, § 29), Danielsson (1968, ch. 3.2), van Fraassen (1972, sect. II), Hansson (1969, sect. XIV), Kutschera (1974, sect. 4), Lewis (1973, ch. 5.1), (1974, sections IV and VI), and Spohn (1975, sect. 1.3). However, a systematic discussion of the possible answers, and of their respective pros and cons, does not exist.

The reader should be warned that, aiming at a survey of the major options and their merits, this paper will have to contain some observations that are neither new nor deep. Furthermore, no glorious winner will emerge from the inquiry, and the logician's choice is limited to various combinations of disadvantages. It seems that the agent's calamity allows for nothing but calamitous theories: When the best options are lacking, then so are flawless accounts of the lack. We, here, concentrate on the deontic, rather than the prohairetic, case; translating "it ought to be the case that" into "individual *a* wants it to be the case that", and "at least as good as" into "desired by *a* at least as much as", the reader can go through the same moves for the logic of preference as the paper explicitly does for the logic of obligation. Within the realm of deontic logics, we concentrate on monadic systems; but since an interpretation of the dyadic "ought" is, in a sense, a collection of interpretations of the monadic "ought", friends of dyadicity will agree that our thoughts carry over to their systems in an obvious way.

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* I am grateful to: Uwe Bombosch, for many discussions on deontic logic; Ulla Wessels and Seán Matthews, for various comments; Thomas Fehige, for drawing a fried egg; everybody involved in running the Center for Philosophy of Science at the University of Pittsburgh, a stimulating and hospitable place where most of this paper has been written; and to all those who commented on oral proto-types of this paper in Pittsburgh and Saarbrücken.

Our reflections are semi-formal and pretty general. No need, then, to recite the definitions of language, well-formed formulae, interpretations, validity, etc. with the usual care. Suffice it to say that we have some run-of-the-mill language of predicate logic, including quantifiers ("$\exists$" for "there is", "$\forall$" for "all", which we will also use as shorthand, even outside formulae) and sentential connectives (like "$\neg$" for "it is not the case that" and "$\rightarrow$" for "if, then"). Furthermore, we have an "O" that, prefixed to any formula A, yields another formula, O(A), to be read as "It ought to be the case that A". Formal bits and pieces can serve as their own names. Possible worlds should be thought of as something like interpretations of classical predicate logic. For a possible world α from a model $\mathscr{F}$ and a formula A, "$\models^{\mathscr{F}}_{\alpha}$ A" means "it is the case in world α that A", and "$\models^{\mathscr{F}}$ A" means "it is the case in $\mathscr{F}$ that A". A[x] refers to an entity that, prefixed by "$\forall$ x", would make a formula. "iff" is short for "if and only if".

Here, then, is the problem.

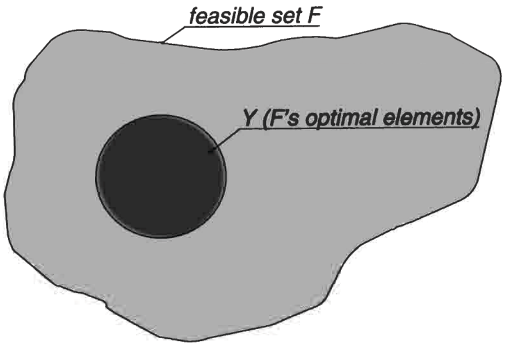
> *Definition (D 1):*
> Let F be a non-empty set of possible worlds. (F is to be thought of as the feasible set, i.e. the set of worlds that, in a given situation, could be brought about; sometimes we will simply refer to F itself as a *situation*.)
> Let $\geq_{\heartsuit}$ be a binary relation on F that is (i) linear, (ii) reflexive and (iii) transitive; that is, we have, for all α, β, $\gamma \in F$: (i) $\alpha \geq_{\heartsuit} \beta$ or $\beta \geq_{\heartsuit} \alpha$, (ii) $\alpha \geq_{\heartsuit} \alpha$, and (iii) if $\alpha \geq_{\heartsuit} \beta$ and $\beta \geq_{\heartsuit} \gamma$, then $\alpha \geq_{\heartsuit} \gamma$. ($\geq_{\heartsuit}$ is to be thought of, and read, as "at least as good as".)
> $\mathscr{F} := \langle F, \geq_{\heartsuit} \rangle$
> $Y_{\mathscr{F}} := \{\alpha \in F \mid \forall\, \beta \in F\ \alpha \geq_{\heartsuit} \beta\}$. (Most of the time we will omit the index "$_{\mathscr{F}}$".)

$\mathscr{F}$, we can say (for today's purposes), is a model of deontic logic. The whole thing then looks like a fried egg, with Y (the set of $\geq_{\heartsuit}$-maximal elements of F) as its yolk (see drawing on next page).

Now, what ought to be the case in situation F? A good first stab would seem to be: Whatever is the case in all of F's optimal worlds. In other words, let's go for the egg's yolk:

(1) $\models^{\mathscr{F}}$ O(A) iff $\forall\, \alpha \in Y \models^{\mathscr{F}}_{\alpha}$ A

But if the feasible set F is infinite, we might be able to find, for every possible world in F, a better one that is also in F. The yolk, then, is



empty. In that case, (1) is unsatisfactory: For every state of affairs A (1)'s right hand side will be true (vacuously so), and hence everything, contradictions included, will be obligatory in situation F.

The reverse problem arises if we take a non-emptiness requirement into (1)'s right hand side, thus getting

(2) $\vDash^{\mathscr{F}} \mathrm{O(A)}$ iff (Y is non-empty and $\forall\, \alpha \in Y \vDash^{\mathscr{F}}_{\alpha} \mathrm{A}$).

For the non-empty yolk, (2) does the same job as (1). But in case the yolk is empty (2) says that nothing whatsoever is obligatory. This is not quite as inadequate as (1), since it does not involve us in deontic contradictions. But it still is inadequate. At the very least we want to be able to tell the agent, say, that she ought not to go for the worst world (if there is one), or some such thing. We don't want to say, as (2) forces us to, that it just doesn't matter what she does. Infinite opportunities for improvement are hardly a moral vacuum.

We can, of course, keep (1) and define the embarrassing case away. Thus proposal (3):

(3) (1)
plus
The Limit Assumption (LA): Every feasible set has an optimal element.

(LA) could simply be written into definition (D 1): "Let F be a set of possible worlds, and $\geq_{\heartsuit}$ a reflexive, linear and transitive relation on F *that has at least one* $\geq_{\heartsuit}$*-maximal element*, [etc.]." The term "Limit Assumption" is due to Lewis (1973). However, unlike Åqvist (1987, § 30), Kutschera (1974, sect. 4) and Spohn (1975, section 1.3), Lewis does not accept (LA), cf. his (1973, ch. 5.1) and (1974, sect. VI).

With (3) we get a deontic logic that simply refuses to comment on the case of the empty yolk. But, since that case *can* exist, we should question the adequacy of such a logic.

Surely the idea behind (3) shouldn't be that we simply exclude all *infinite* feasible sets — a proposal that would entail (LA). After all, infinity is with us all the time. (There are, for example, infinitely many ways of drawing a line on a given piece of paper.) From the stronger proposal back to (LA) itself, then. Our ideal of a *complete* moral theory is that of a theory that applies to *all conceivable* situations. But it is easy to conceive of stories that involve non-optimality. Suppose John lives in a world in which infinitely many people are starving, but for some strange reason, if he starts pronouncing any natural number *n*, *n* people will be relieved from their suffering and lead a happy life ever after. Obviously, John's feasible set has no best element. You may now say that if John's life is finite, then so is the set of natural numbers that come to his mind; fine, but what if John is immortal? This, you may reply, is all science fiction. Yes, but logic should be prepared for it, and it is wise of philosophers, including moral philosophers, to spend half of their time discussing unreal cases in order to attain clarity in down-to-earth cases.

Another example of a widespread doctrine that involves non-optimality for certain cases concerns 'different people choices': Is there a degree of happiness such that the more people of at least that degree of happiness come into existence the better? Many moral philosophers (among them B. Anglin, J. Glover, R. M. Hare, Y.-K. Ng, T. G. Roupas, R. Sikora, J. J. C. Smart) answer in the affirmative. Again, it would follow that the set of all possible worlds and many of its infinite subsets do not have optimal elements.

Next, take the quest for a theodicy: What ought an omnipotent being to do? Especially, ought she to create anything like the world we live in? One of the most powerful moves that has been made in that field is George Schlesinger's (cf. his (1977), chapters 9 and 10): Schlesinger suggests that God's feasible set, which is certainly infinite, would not contain an optimal element. Thus, for logical reasons (hence for reasons that don't impugn her omnipotence), God would have to do something sub-optimal. Following (3), the logic of ethics would loose out on one of ethics' all time favourites.

For many moralities, then, (3) would force us to give up the applicability of deontic logic to a large class of feasible sets, including the set of *all* possible worlds. There is also a structural, conceptual, reason why that loss of scope would be regrettable: It implies that "ought, *ideally*" would no longer be a special case of our logic, since we get "ought, *ideally*" precisely if we let loose "ought" on an unconstrained feasible set, i.e. on the set of all possible worlds. Of course, if a deontic logic just couldn't say anything reasonable about feasible sets without optimal elements, then we would have to live with such a limited logic. But, as the foregoing observations show, the restriction is unwelcome, and worth overcoming if possible.

Let us go on trying, then, and consider a proposal of Kutschera's:

(4) (1),
but redefine $Y := \{\alpha \mid \alpha \text{ is better than the actual world}\}$

(4) rephrases the semantics for system D that Kutschera gives in (1973), ch. 1.9, but that he now rejects (1991, in conversation). The idea here, one could say, is to replace bestness by betterness, and to 'satisfice' rather then to optimize. There are quite a few problems with this system. Some, but, as will be seen in our discussion of (5), not all, of them, are concerned with why you should choose, of all worlds, the *actual* world as a moral standard. Firstly, there is little point in asking what you ought to do if you already know which world is the actual one. If you have a moral problem, ethics should hardly tell you: "*First, look at the actual world*, then look at all the worlds better than the actual one, [etc.]"; since if you know which world is actual, what space is left for decision? How can your feasible set have more than one element (viz. the actual world)? And how could we have a *moral* problem where we don't even have a *decision* problem?

Secondly, suppose there is a state of affairs A true in all of (4)'s Y-worlds but not in the actual world, so that A ought to be brought about. Fulfilling the obligation means, therefore, that a world other than the actual world would have to be actualized, which is a difficult thing to achieve.

Thirdly, suppose there are optimal elements in F, and you will do the right thing, i.e. actualize one of them. It then follows that Y, as (4) characterizes it, is empty. But Kutschera doesn't want it to be empty (cf. (1973), p. 52), and quite rightly so. For if Y were empty, (1)'s right hand side would again be vacuously true for every A, and hence everything, contradictions included, would be obligatory. So Y *mustn't* be empty, but doing the right thing would *make* it empty.

Even if we leave these somewhat structural complications aside, what could be a reason for giving so much *normative* weight to the actual world as such? Morally speaking, actuality is an entirely arbitrary standard. (With the obvious exception of what is actual 'already' and thus not subject to choice anymore. But that actuality is just a constraint on the feasible set itself, and doesn't give moral authority to one, rather than another, element *of* the feasible set.)

Fifthly, let me illustrate an appalling consequence of this logic. In the actual world, every day about 35 000 children under five die of malnutrition. (FAO, 1992, p. 3) Let us say, for short, that they *starve*. Now suppose it were in our hands to bring about either a world β in which 34 000 children starve every day or a world α in which nobody starves anymore. Let other things in α and β be equal, so that α is better than β and both of them are better than the actual world:

α	Nobody starves.
$\vee_{\heartsuit}$	
β	34 000 children starve per day.
$\vee_{\heartsuit}$	
actual world	35 000 children starve per day.

Though we could bring it about, and though it involves no moral drawbacks, it is, according to (4), not the case that nobody ought to starve. And the *reason* for this is that, though the feasible set contains an optimal world, it also contains a suboptimal world, better than the actual one, in which people starve! Saving 34 000 lives would be 'supererogatory'. This is absurd. The system simply gives us too few obligations. In general, it lets us get away with the tiniest improvements. Of any more serious feasible improvement (no matter how good or important it may be) the system cannot say that we ought to effect it. Note that this inadequacy already arises in finite feasible sets, hence in feasible sets *with* optimal elements.

At this point, one might be tempted to banish actuality from the definition of Y while keeping (4)'s structure. One could simply choose another world (the cut-off point, as we shall call it), to play the part of the moral standard, and define Y as the set of worlds at least as good as that world:

(5) (1),
but redefine $Y := \{\alpha \mid \alpha \geq_{\heartsuit} \text{cut-off point}\}$

So, in order to get your semantics going, you need a third input item. In addition to F and $\geq_{\heartsuit}$ (the two ingredients of $\mathscr{F}$ that our initial definition

(D 1) mentions so far), you need to distinguish one element of F as the cut-off point. In a case where you *have* a real yolk you would of course take any of the yolk worlds as your cut-off point. Your semantics would then work, for those cases, exactly like (1), and that's good.

But in cases of non-optimality the unhappy type of results remains intact. Here is one of the examples from above:

$$
\begin{array}{lllllllllllll}
\cdot & & & & & \dots & & & & & & \\
\cdot & & & & & \dots & & & & & & \\
\cdot & & & & & \dots & & & & & & \\
\alpha_{i+1} & R(a_1) & R(a_2) & R(a_3) & R(a_4) & \dots & R(a_i) & R(a_{i+1}) & \neg R(a_{i+2}) & \dots & \neg R(a_j) & \dots \\
\vee\blacktriangledown & & & & & & & & & & & \\
\alpha_i & R(a_1) & R(a_2) & R(a_3) & R(a_4) & \dots & R(a_i) & \neg R(a_{i+1}) & & \dots & \neg R(a_j) & \dots \\
\vee\blacktriangledown & & & & & & & & & & & \\
\cdot & & & & & & & & & & & \\
\cdot & & & & & & & & & & & \\
\cdot & & & & & & & & & & & \\
\vee\blacktriangledown & & & & & & & & & & & \\
\alpha_3 & R(a_1) & R(a_2) & R(a_3) & \neg R(a_4) & & & & & \dots & \neg R(a_j) & \dots \\
\vee\blacktriangledown & & & & & & & & & & & \\
\alpha_2 & R(a_1) & R(a_2) & \neg R(a_3) & & & & & & \dots & \neg R(a_j) & \dots \\
\vee\blacktriangledown & & & & & & & & & & & \\
\alpha_1 & R(a_1) & \neg R(a_2) & & & & & & & \dots & \neg R(a_j) & \dots
\end{array}
$$

We have a feasible set that contains, for every natural number i, a world α_i . All these worlds have the same inhabitants, viz. the persons a_1, a_2, and so on. In world α_1 (bottom line), only individual a_1 is relieved (we use "R" for "is relieved"), and all the others starve; in world α_2 (second line from the bottom), only a_1 and a_2 are relieved, and so on. Let, for every i, world α_{i+1} be better than world α_i.

If in this situation you choose, say, α_i as the cut-off point, then it it is not true, according to (5), that any of the people a_{i+2}, a_{i+3}, etc. ought to be relieved. But where to draw a non-arbitrary line? How do you explain to person a_{i+2} that you have an obligation to save her colleague a_{i+1}, but no obligation to save *her*? Such a solution levels all moral differences above α_i. As far as your deontic logic and your obligations are concerned, there is no difference between α_{i+1} and $\alpha_{i+3\,000\,000}$. Millions of avoidably starving people just don't occur in your morality anymore. Above α_i, it's all the same to you. So (5), too, seems to give us too few obligations.

There is also another problem. Suppose you want to look at different feasible sets from the point of view of one moral system, i.e. from the same "at least as good as"-relation over all possible worlds. In the above example of a feasible set F with an *empty* yolk, let α_i be again the name of your cut-off point. We said that if a set's yolk is non-empty one would choose a yolk-world as the cut-off point. (Since with a non-empty yolk there is no excuse for having the logic generate fewer obligations than that; no excuse, that is, for replacing bestness by betterness.) The F under consideration will have a finite subset whose optimal element (and hence cut-off point) is, say, α_{i+2000}.

This gives us a suspicious sort of 'non-monotony' (as one might call it): Though the subset is smaller than the original set, its standards are higher. In the subset, you have to be as good as α_{i+2000}, in the big feasible set you have all the worlds from the subset — but suddenly being as good as α_i suffices. "If you can save $i+2000$ lives and no more, then save $i+2000$ lives", says (5), "but if you can save as many lives as you wish, then saving i of them will do." This is bizarre. (Here is another way of putting what went wrong: Reading "C" as the operator that, applied to feasible sets, forms their yolks, we have violated the requirement of rationality that Sen calls "Property α": "$x \in S_1 \subset S_2 \rightarrow [x \in C(S_2) \rightarrow x \in C(S_1)]$, for all x" (1970, ch. 1.6).) So (5), too, has serious flaws.

Next, consider the following way out:

(6) $\models^{\mathcal{F}} O(A)$ iff $\exists\, \beta \in F\ \forall\, \alpha \in F\ (\alpha \geq_{\heartsuit} \beta \Rightarrow\ \models^{\mathcal{F}}_{\alpha} A)$

(6) rephrases the spirit of the solution that can be found at the beginning of Lewis (1971), in section II of van Fraassen (1972), sections 1.3, 1.4 and 5.1 of Lewis (1973), and in sections IV and VI of Lewis (1974). If you have an optimum, then (6), like many of its alternatives, does the same as (1): no problem. But if your worlds go on getting better and better, then (6) says: If somewhere on the ladder of betterness there is a rung above which all the worlds are A-worlds, then A ought to be the case. This looks like the natural and happy supplement to the optimal subset approach for those cases where we don't have an optimal subset. It respects the optimal elements where they exist and has something to say where they don't exist. It also gives us the obligations that system (5) did not want to: in the starvation case it is now, for every natural number i, obligatory to save individual a_i.

(For the sake of completeness, a brief warning. The temptation to swap the quantifiers in (6) should be resisted; it would yield

(7) $\models^{\mathcal{F}} O(A)$ iff $\forall\, \beta \in F\ \exists\, \alpha \in F\ (\alpha \geq_{\heartsuit} \beta \Rightarrow\ \models^{\mathcal{F}}_{\alpha} A)$,

and it is easy to conceive of models in which both for a sentence B and for its negation $\neg$B (7)'s right hand side gets true, so that (7) would generate deontic contradictions. Back to (6), then.)

Unsurprisingly, (6), too, has its problems; for example with respect to "'ought' implies 'can'". Let us disambiguate that principle. The *Strong* "'ought' implies 'can'" Principle, (SOC), reads as follows:

(SOC) $\exists\, \alpha \in F\ \forall\, \mathrm{A}\ (\models^{\mathcal{F}} \mathrm{O}(\mathrm{A}) \Rightarrow\ \models^{\mathcal{F}}_{\alpha} \mathrm{A})$.

(SOC) says that it is possible to jointly fulfil all the obligations one has. The *Weak* "'ought' implies 'can'" Principle, (WOC), asserts the same for any finite number of the obligations one has:

(WOC) $\forall$ finite subsets B of $\{\mathrm{A} \mid\ \models^{\mathcal{F}} \mathrm{O}(\mathrm{A})\}\ \exists\, \alpha \in F\ \forall\, \mathrm{A} \in B\ \models^{\mathcal{F}}_{\alpha} \mathrm{A}$.

Obviously, models of system (6) always respect (WOC), but, and this is a drawback, they don't always respect (SOC). (Maybe the drawback would be less severe if, instead of a logic for "ought, all things considered" we were designing a logic for "good, all things considered" or "wanted, all things considered"; it seems less annoying if the latter two categories fall short of the feasible than if obligations do.)

A related feature of (6) is worth mentioning:

($\forall$) $\quad \forall\, x\ \mathrm{O}(\mathrm{A}[x]) \rightarrow \mathrm{O}(\forall\, x\ \mathrm{A}[x])$

is not valid in system (6). Look again at the relief matrix we considered when we started discussing (5), and suppose that for every individual in the domain there is a natural number n such that a_n is that individual's name. In that example, we then have, according to (6), all the $\mathrm{O}(R(x))$, but we don't have $\mathrm{O}(\forall\, x\ R(x))$. Furthermore, we can extend the example to make it illustrate why ($\forall$) *should* not be valid in (6), and, thus, why we should not even be tempted to reform (6) in that respect; suppose we add the following bottom line α_0 to the relief matrix:

$$\begin{array}{llllll} \alpha_1 & \ldots & & & & \\ \vee\!\blacktriangledown & & & & & \\ \alpha_0 & R(a_1) & R(a_2) & \ldots & R(a_j) & \ldots \end{array}$$

For God knows which reason (perhaps somebody must suffer in order to keep up the world's spiritual link to its saviour, or whatever), *everybody's* having property R is the worst thing that could happen. We have a clear case here in which ($\forall$) both *is* and, given the spirit of solution (6), *should* be false. The case also shows that in some models of (6) an agent who fulfilled a certain subset of her obligations would ipso facto have to violate some other obligation of hers. (Suppose she fulfilled all the $\mathrm{O}(R(x))$; she would then violate $\mathrm{O}(\neg \forall\, x\ R(x))$.) One might consider this to be tolerable for things like prima facie obligations, but intolerable for the final "ought" that deontic logics (of the style we are studying here) want to capture. However, the phenomenon is less like a deontic contradiction than it appears to be: Note that in the relevant models the obligations whose joint fulfilment *would* violate another obligation *cannot* be jointly fulfilled. (In our case: the agent cannot fulfil all the $O(R(x))$.)

For propositional logics, David Lewis has shown (1973, sect. 6.1) that the choice between (3) and (6) does not affect the set of valid formulae (the set of formulae true in every model). There are various reasons why that shouldn't stop us from putting some intelligence into the choice. The most general reason is that there is more to a proposed definition of "model" then providing the right class of valid sentences (of sentences true in all models, that is). It must make sense at the lower level, too, where it assigns truth conditions to particular sentences (that may not be logical truths) in particular models. If it gives, in a model, truth conditions to "John ought to hit Peter", then these must make sense, and they might fail to do so *though* the system generates the adequate class of *valid* sentences. A second, less general reason is that, as we saw in the discussions of (3) and (6), the choice between the two affects a number of issues in moral philosophy: the logic's scope, different people choices, theodicy, "ought, ideally", "'ought' implies 'can'", and others. And finally, (∀), valid in (3) but not in (6), shows us that Lewis's result will not carry over to the realm of deontic *predicate* logic.

Next, let me point out an annoying feature of both (5) *and* (6), the problem of 'nihilistic spirals': There can be feasible sets without optimal elements in which (5) and (6) yield nothing but tautological obligations. Things could be better and better and better, but without obligations to make them so.

The proof runs as follows. Let $A_1, \ldots, A_i, \ldots$ be an enumeration of all the non-contradictory formulae of (classical) predicate logic. (The proof would work equally well with propositional logic.) For any i, let $((A_i)_j)_{j\in\mathbb{N}}$ be a family of sets $(A_i)_j$ such that: for every j, $(A_i)_j$ is a maximal consistent (=: max. cons.) set of formulae of predicate logic, contains the formula A_i, and is, for all $k \neq j$, different from $(A_i)_k$. (It is easy to see that such a family will always exist, i.e. that there are infinitely many different max. cons. sets (A_i) that contain A_i.)

Now, let for all $n \in \mathbb{N}$ the function g be defined recursively as follows:

$$g(1) = 1$$

$$g(n+1) = \begin{cases} g(n) + 1 & \text{if } n \neq 1 \text{ and} \\ & g(n) \leq max\{g(m) \mid m \in \mathbb{N} \;\&\; m \leq n-1\} \\ 1 & \text{otherwise} \end{cases}$$

g yields the sequence 1, 1,2, 1,2,3, 1,2,3,4, 1,2,3,4,5, etc. No matter where in this sequence you are, every natural number will turn up at a later stage; we can refer to this property as g's "prospective completeness".

Let an enumeration f of max. cons. sets be defined recursively as follows:

$$\begin{aligned} f(1) &= (A_1)_1 \\ f(n+1) &= (A_{g(n+1)})_{min\{i \in \mathbb{N} \mid \forall j \in \mathbb{N}\, (j \leq n \Rightarrow f(j) \neq (A_{g(n+1)})_i\}} \end{aligned}$$

It is enumeration f we shall have in mind when speaking of the "first", "second", "third" etc. max. cons. set.

Whatever your modal ontology may look like, either a max. cons. set of sentences *is* itself a possible world, or there is a possible world in which the sentences of that set *are true*. I will use the latter terminology (but nothing hinges on the decision). Thus, for every set that f enumerates we can find a possible world such that all the sentences in the set are true in that world. Let α_1 be any world that does this for the first set, α_2 one that does it for the second, etc. $F := \{\alpha_i \mid i \in \mathbb{N}\}$; $\alpha_i >_{\blacktriangledown} \alpha_j := i > j\ (\forall\, i, j \in \mathbb{N})$.

Let B be any logically contingent formula of predicate logic (i.e. one that is neither a tautology nor a contradiction). Suppose that

(S) according to (5) or (6), $\models^{\mathcal{F}}$ O(B).

There are $i, j \in \mathbb{N}$ such that, in our initial enumeration of formulae, B is the formula A_i and $\neg$B the formula A_j. It follows from (S) that

(S′) there is a $k \in \mathbb{N}$ such that $\forall\, l \in \mathbb{N}$: $(l \geq k \Rightarrow \models^{\mathcal{F}}_{\alpha_l} B)$

(If we move within (5), we will have a cut-off point, say α_p. Then p would be a k with the required property.)

But g is prospectively complete. So there will be a $q \in \mathbb{N}$ such that $q > k$ and $g(q) = j$. Let us look at any such q. It follows from the definition of f that the q-th max. cons. set contains A_j, i.e. $\neg$B, and therefore we have $\models^{\mathcal{F}}_{\alpha_q} \neg$ B. But $q > k$, hence (S′) is false, hence (S) is false: According to systems (5) and (6), *nothing* (other than tautologies) is obligatory in situation F.

Let me sketch, in a more impressionist way, what has happened here. If $A_1, \ldots, A_i, \ldots$ is an enumeration of the non-contradictory formulae of predicate logic, and if, for every i, $((A_i)_j)_{j \in \mathbb{N}}$ is a family of pairwise distinct maximal consistent sets each of which contains the sentence A_i, then we can get a 'list' (or spiral) of the following type (enter it at the bottom, at α_1, and work your way up; the sentences of a listed set are all true in the possible world α_i that is listed on the same line as the set):

.				.	
.				.	
.				.	
α_{10}				$(A_4)_{s_{10}}$	
$\vee_\heartsuit$					
α_9				$(A_3)_{s_9}$	
$\vee_\heartsuit$					
α_8				$(A_2)_{s_8}$	…
$\vee_\heartsuit$					
α_7				$(A_1)_{s_7}$	
$\vee_\heartsuit$					
α_6			$(A_3)_{s_6}$		
$\vee_\heartsuit$					
α_5			$(A_2)_{s_5}$	(where s_5 is the smallest s such that etc.)	
$\vee_\heartsuit$					
α_4			$(A_1)_{s_4}$	(where s_4 is the smallest s such that the set $(A_1)_s$ is not yet on the list)	
$\vee_\heartsuit$					
α_3		$(A_2)_{s_3}$	(where s_3 is the smallest s such that the set $(A_2)_s$ is not yet on the list)		
$\vee_\heartsuit$					
α_2		$(A_1)_{s_2}$	(where s_2 is the smallest s such that the set $(A_1)_s$ is not yet on the list)		
$\vee_\heartsuit$					
α_1	$(A_1)_1$				
$\uparrow$					
$\mathscr{F}$					

For any contingent formula A, both A and $\neg$ A will have been enumerated in our initial enumeration of formulae. So for every possible world α_i we can find worlds α_j and α_k such that: $\alpha_j >_\heartsuit \alpha_i$, and $\alpha_k >_\heartsuit \alpha_i$, and α_j 'makes true' a set $(A)_s$ (for some natural number s), and α_k 'makes true' a set $(\neg A)_t$ (for some natural number t). By definition, $A \in (A)_s$ and $\neg A \in (\neg A)_t$; thus, for every contingent formula A and every natural number i we can find worlds α_j and α_k such that: $\alpha_j >_\heartsuit \alpha_i$, and $\models^{\mathscr{F}}_{\alpha_j} A$, and $\alpha_k >_\heartsuit \alpha_i$, and $\models^{\mathscr{F}}_{\alpha_k} \neg A$.

Thus, we have generated an infinite feasible set F in which the worlds get better and better but in which neither (5) nor (6) come up with *any* obligations (other than those to make tautologies true). Note that this amounts, practically, to a special case of a shortcoming that (right at the beginning of this paper) we said discredited one approach to the problem of empty yolks: We blamed suggestion (2) for implying that in cases of empty yolks nothing is obligatory. We now know that the troublesome lack of obligations that plagues (2) in *all* cases of empty yolks plagues (5) and (6) in *some* cases, too. It looks, therefore, as if (5) and, surprisingly, the much-favoured (6), were at best in pretty much the same class as (2).

There are two ways we can steer (5) and (6) clear of trouble. Firstly, we could pay the price of allowing for formulae of infinite length. We can then look, for every max. cons. set A that is a value of f, at the conjunction of all the sentences in A. Let us call that formula A's conjunction. For every f-value A, (6) would give us as obligatory the negation of A's conjunction, and, analogously, for every finite number of such sets, (6) would say that the conjunction of the negated conjunctions of those sets is obligatory. (5) would say that the negated conjunctions of the sets below the cut-off point, as well as any conjunctions of those negated conjunctions, are obligatory.

Secondly, we might try to characterize, and then to exclude from the logic's scope, the nihilistic spirals that are the backbone of our proof. After all, they are somewhat bizarre. The moralities imply that it is not facts that carry any value, just infinite collections of facts. To exclude the freakish cases, we could, in our initial definition (D 1), require the existence of a world α in F and of a contingent formula A such that A is the case in all those of F's worlds that are at least as good as α. We could even require, for *every* contingent formula A, the existence of a world α in F such that A is true in all those of F's worlds that are at least as good as α. But we would be pressed hard to find a *plausible* requirement for the purpose: a demarcation that, on the one hand, does not rule out too many models, and, on the other, does not continue to include models in which the system yields too few obligations. Apart from that, denying modelhood on grounds of freakishness would amount, like the Limit Assumption itself, to a dent in the scope or in the moral neutrality of deontic logic.

Time to sum up: What can we choose from? Systems (1), (4) and (7) are not candidates. The choice among the remaining options comes down to a choice among combinations of the following six bitter pills (A) to (F):

(A) The yolk can be empty, and if it is, then nothing is obligatory.

(B) The system is not applicable to all feasible sets in all moralities.

(C) The system exhibits a somewhat irrational sort of 'non-monotony'. (It violates Sen's "Property α".)

(D) (SOC) may be violated: Maybe you cannot jointly fulfil all your obligations.

(E) There can be cases of empty yolks in which nothing is obligatory that is not a tautology.

(F) Infinitely long formulae are allowed.

You have to swallow one of the bitter pills (A), (B), (C) and (D). If you accept (A), then you accept (E), for (E) is entailed by (A). And if you choose to swallow (C) or (D), you have to swallow either (B) or (E) or (F) in addition:

Option …	… has disdvantage(s) …
(2)	(A)
(3)	(B)
(5)	(C) & (E)
(5), plus infinitely long formulae	(C) & (F)
(5), plus exclusion of bizarre models	(C) & (B)
(6)	(D) & (E)
(6), plus infinitely long formulae	(D) & (F)
(6), plus exclusion of bizarre models	(D) & (B).

Take your pick.

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